Comment on “Viscous cosmology in the Kasner metric”.

Mauricio Cataldo [a b] * and Sergio del Campo [c] †

[a] *Departamento de Física, Facultad de Ciencias, Universidad del Bío-Bío, Avda. Collao 1202, Casilla 5-C, Concepción, Chile.*

[c] *Instituto de Física, Facultad de Ciencias Básicas y matemáticas, Universidad Católica de Valparaíso, Avenida Brasil 2950, Valparaíso, Chile.*

Abstract: We show in this comment that in an anisotropic Bianchi type I model of the Kasner form, it is not possible to describe the growth of entropy, if we want to keep the thermodynamics together with the dominant energy conditions. This consequence disagrees with the results obtained by Brevik and Pettersen [Phys. Rev. D **56**, 3322 (1997)].

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Brevik and Pettersen [1] have studied the consequences that a Bianchi type I metric of the Kasner form

$$ds^2 = -dt^2 + t^{2p_1}dx^2 + t^{2p_2}dy^2 + t^{2p_3}dz^2 \tag{1}$$

occurs for the equation of state for the cosmic fluid, characterized by a shear viscosity η and a bulk viscosity ξ. In their work, they concluded that for a viscous fluid, with $\eta \neq 0$ and $\xi \neq 0$, and from Einstein equations, the requirement that the three Kasner parameters p_i $(i = 1, 2, 3)$ be constant implies that $\eta \sim 1/t$ and $\xi \sim 1/t$.

From their equation (29) it is obtained an explicit expression for the shear viscosity given by

$$\eta = \frac{1}{2\kappa t}(1 - S), \tag{2}$$

where $\kappa = 8\pi G$ and $S = \sum_{i=1}^{3} p_i$. It could be shown from thermodynamics that we should impose the condition $\eta \geq 0$ [2]. Therefore, we see from expression (2) that we must require that

$$S - 1 \leq 0. \tag{3}$$

On the other hand, the entropy production, in an anisotropic Kasner type universe, becomes

$$\dot{\sigma} \approx \frac{2S^2}{nk_{_B}Tt^2}\eta A, \tag{4}$$

where $A = 1/3\sum_{i=1}^{3}(1 - H_i/H)^2 = 3Q/S^2 - 1 \geq 0$, $Q = \sum_{i=1}^{3} p_i^2$, n is the baryon number density, $k_{_B}$ is the Boltzmann constant and T the temperature.

In expression (4) we have restricted to the case in which the shear viscosity η is vastly greater than the bulk viscosity ξ, as was considered in Ref. [1].

Thus, following the result obtained by Brevik and Pettersen, we conclude that in an anisotropic Kasner type model, the parameters entering in the metric have to satisfy the bound $S \leq 1$, in order to deal with an appropriated physical model.

On the other hand, if we require the model to satisfy the dominant energy conditions, specified by $-\rho \leq P_j \leq \rho$ [3], where ρ is the energy density and P_j (with $j = x, y, z$) are the effective momenta related to the corresponding coordinate axis, we could show that the shear viscosity η necessarily, in this sort of model, becomes negative, since these dominant energy conditions imply that $S \geq 1$, and not $S \leq 1$, as was specified by Brevik and Pettersen.

To see this, let us write the dominant energy condition explicitly in terms of the parameters that enter in the metric (1), i.e. p_1, p_2 and p_3.

Einstein field equations can be written in comoving coordinates as (with $\kappa = 8\pi G = 1$)

$$\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{c}}{c} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = \rho, \tag{5}$$

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = -P_x, \tag{6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\dot{a}}{a}\frac{\dot{c}}{c} = -P_y \tag{7}$$

and

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = -P_z, \tag{8}$$

where a, b and c are the anisotropic expansion factors. From the metric (1) they are given by $a = t^{p_1}$, $b = t^{p_2}$ and $c = t^{p_3}$. Thus, the Einsteins field equations (5)-(8) reduce to

$$\rho = \frac{p_1p_2 + p_1p_3 + p_2p_3}{t^2}, \tag{9}$$

$$P_x = -\frac{p_2^2 + p_3^2 - p_2 - p_3 + p_2p_3}{t^2}, \tag{10}$$

$$P_y = -\frac{p_1^2 + p_3^2 - p_1 - p_3 + p_1p_3}{t^2} \tag{11}$$

*E-mail address: mcataldo@alihuen.ciencias.ubiobio.cl

†E-mail address: sdelcamp@ucv.cl

and

$$P_z = -\frac{p_1^2 + p_2^2 - p_1 - p_2 + p_1 p_2}{t^2}. \quad (12)$$

Here, as was mentioned above, P_j, with $j = x, y, z$, represent the effective momenta in the corresponding coordinate axis. Note that either ρ and P_j (with $j = x, y, z$) scale as t^{-2}. Thus, the dominant energy conditions will give some specific relations between the Kasner parameters p_i.

The conditions $P_j \leq \rho$, with $j = x, y, z$, yield to three inequations given by

$$(S - p_1)(S - 1) \geq 0, \quad (13)$$

$$(S - p_2)(S - 1) \geq 0 \quad (14)$$

and

$$(S - p_3)(S - 1) \geq 0, \quad (15)$$

which, after adding them, reduced to just one inequation given by

$$2S(S - 1) \geq 0. \quad (16)$$

In a similar way, from $P_j \geq -\rho$ we get the inequations

$$S(1 + p_1) - p_1 \geq Q, \quad (17)$$

$$S(1 + p_2) - p_2 \geq Q, \quad (18)$$

and

$$S(1 + p_3) - p_3 \geq Q. \quad (19)$$

After adding them we get

$$S \geq \frac{AS^2}{2}. \quad (20)$$

From expression (20), we see that $S \geq 0$, since by definition $A \geq 0$. With this condition on S, we obtain from expression (16) that necessarily S should be greater than one. This means that a Bianchi type I metric of the Kasner form, always give rise to a negative shear viscosity, where expression (2) applies. From this result, and from expression (4), we observe that we are finished with the unfavored situation in which $\dot{\sigma} \leq 0$, giving the meaning that the entropy in this sort of universe decreases instead of increasing.

In conclusion, we have shown in this comment that it is not possible to describe the growth of entropy in the universe for a viscous anisotropic Bianchi type I metric of the Kasner form, if we want to keep the thermodynamic conditions together with the dominant energy conditions.

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